GPU-Centered Font Rendering Directly from Glyph Outlines

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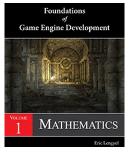
About the speaker

- Working in game/graphics dev since 1994
  - Previously at Sierra, Apple, Naughty Dog
- Current projects:
  - Slug Library, C4 Engine, The 31st, FGED

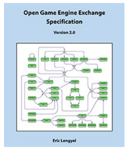

Foundations of Game Engine Development

Volume 1 Mathematics

Eric Lengyel

GAME ENGINE GEMS 3

Edited by Eric Lengyel


Open Game Engine Exchange Specification

Version 2.0

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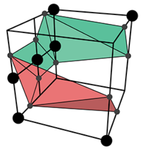


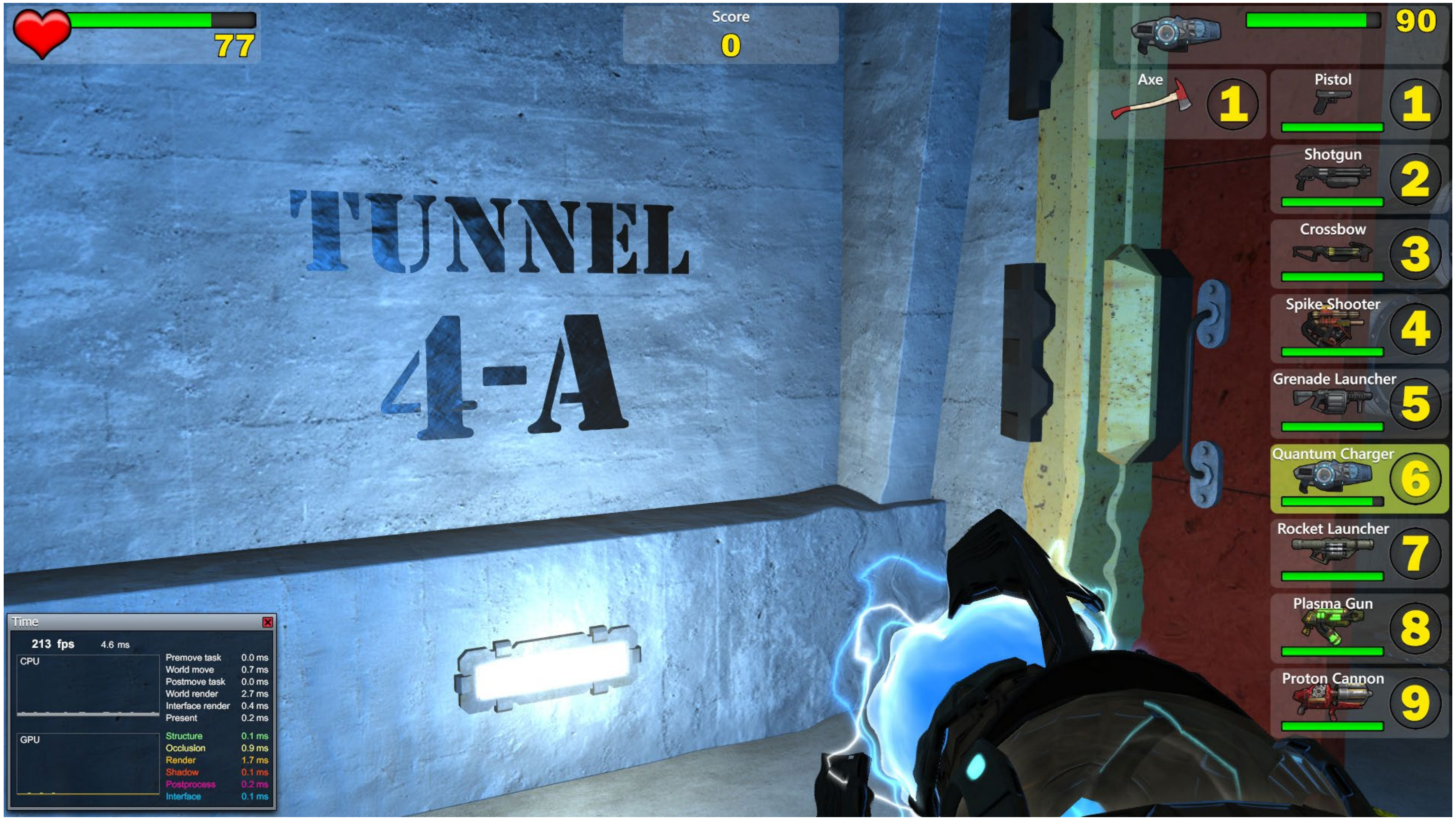
About this talk

- Technical details about the “Slug” font rendering algorithm
- Paper published in JCGT in June 2017
- New developments in past year

Font Rendering Ubiquity

- Text rendered everywhere in 3D applications
  - GUI: Buttons, checkboxes, lists, menus, ...
  - Games: Score, health, ammo, ...
  - In scene: Signs, labels, computer screens, ...
  - Debug info: Console, stats, timings, ...

GPU-Centered Font Rendering Directly from Glyph Outlines



```
213 fps     4.6 ms

Premove task       0.0 ms
World move         0.7 ms
Postmove task      0.0 ms
World render       2.7 ms
Interface render   0.4 ms
Present            0.2 ms

Structure          0.1 ms
Occlusion          0.9 ms
Render             1.7 ms
Shadow             0.1 ms
Postprocess        0.2 ms
Interface          0.1 ms
```

Font Rendering Design Goals

- Unified approach
  - Same technique used to render all text in all situations
- Runs in shader on GPU
  - Fully dynamic, but also allows caching
  - Can be combined with other materials

Font Rendering Design Goals

- Total resolution independence
  - No precomputed images or distance fields
- Ability to render with any transform
  - Arbitrary scale, rotation, perspective
- Must look good at large and small font sizes

Font Rendering Design Goals

- Minimal triangulation
  - Don’t want lots of small triangles
  - GPUs perform best with large triangles
  - A fixed per-glyph vertex count is desirable
  - Would like to be able to easily clip text
  - Would like to apply text to curved surfaces

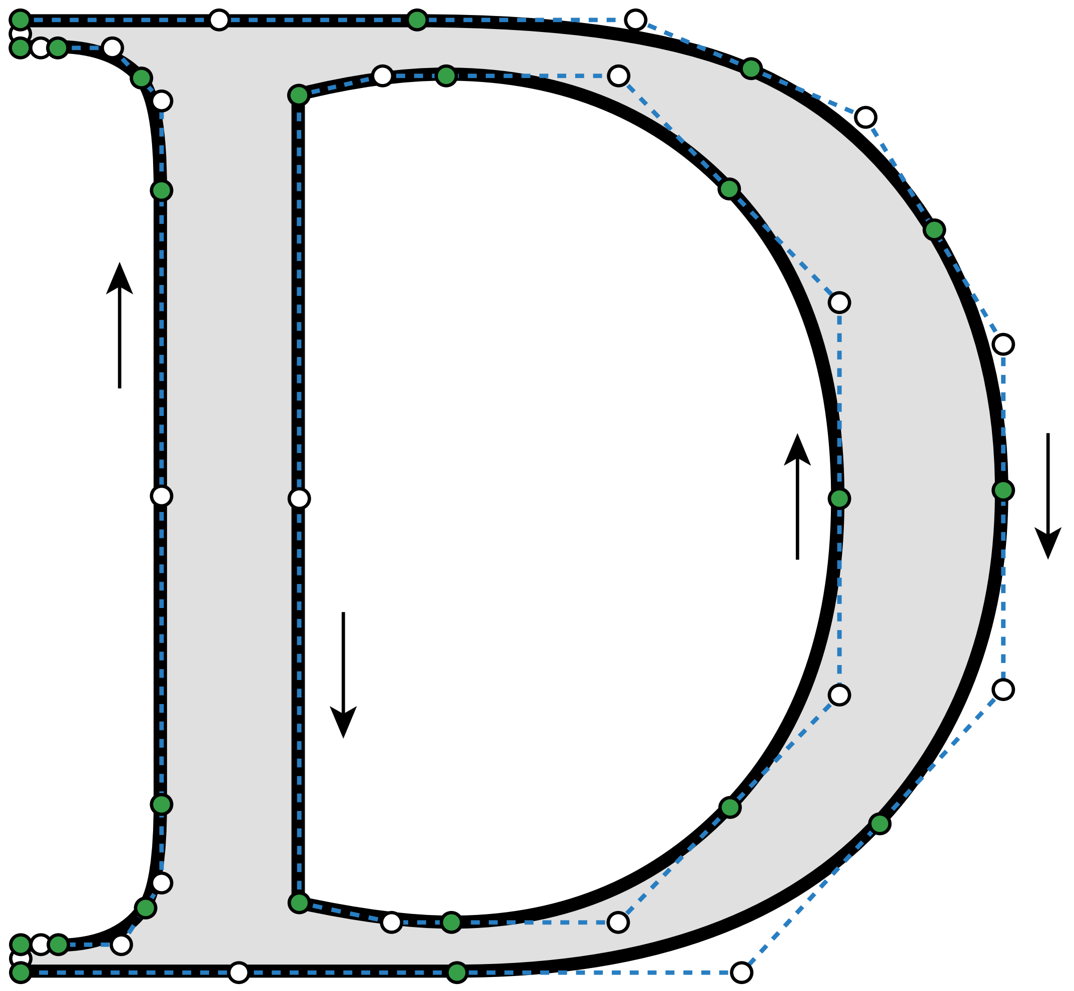
Rendering Algorithm Priorities

1. Works correctly
2. Looks good
3. Runs fast

Glyphs in TrueType

- Glyph defined by one or more closed contours
- Each contour composed of continuous sequence of quadratic Bézier curves
- Each Bézier curve has three control points

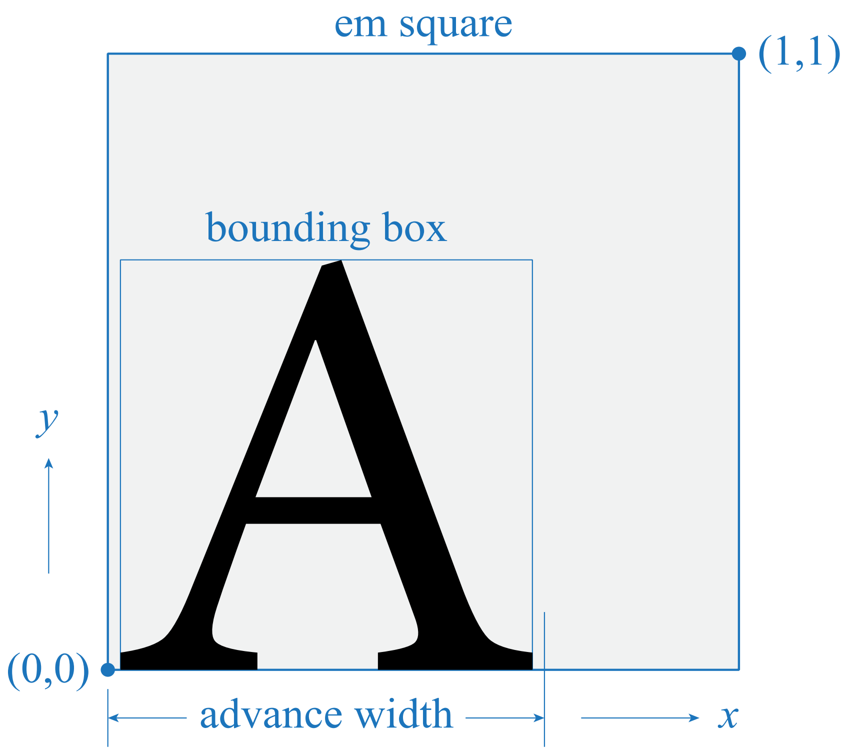
GPU-Centered Font Rendering Directly from Glyph Outlines



Glyph Space

- Glyphs are defined on em square
- Coordinates in range [0,1] inside em square
- Curves can extend outside em square

Glyph Space



Winding Number

- To determine whether point inside glyph, calculate *winding number* with respect to each contour and sum

- Point inside glyph outline if sum of winding numbers is nonzero

Winding Number

- The winding number is the count of complete loops a contour makes around a point
- One direction (arbitrary, either CW or CCW) is considered positive, and opposite direction is then considered negative

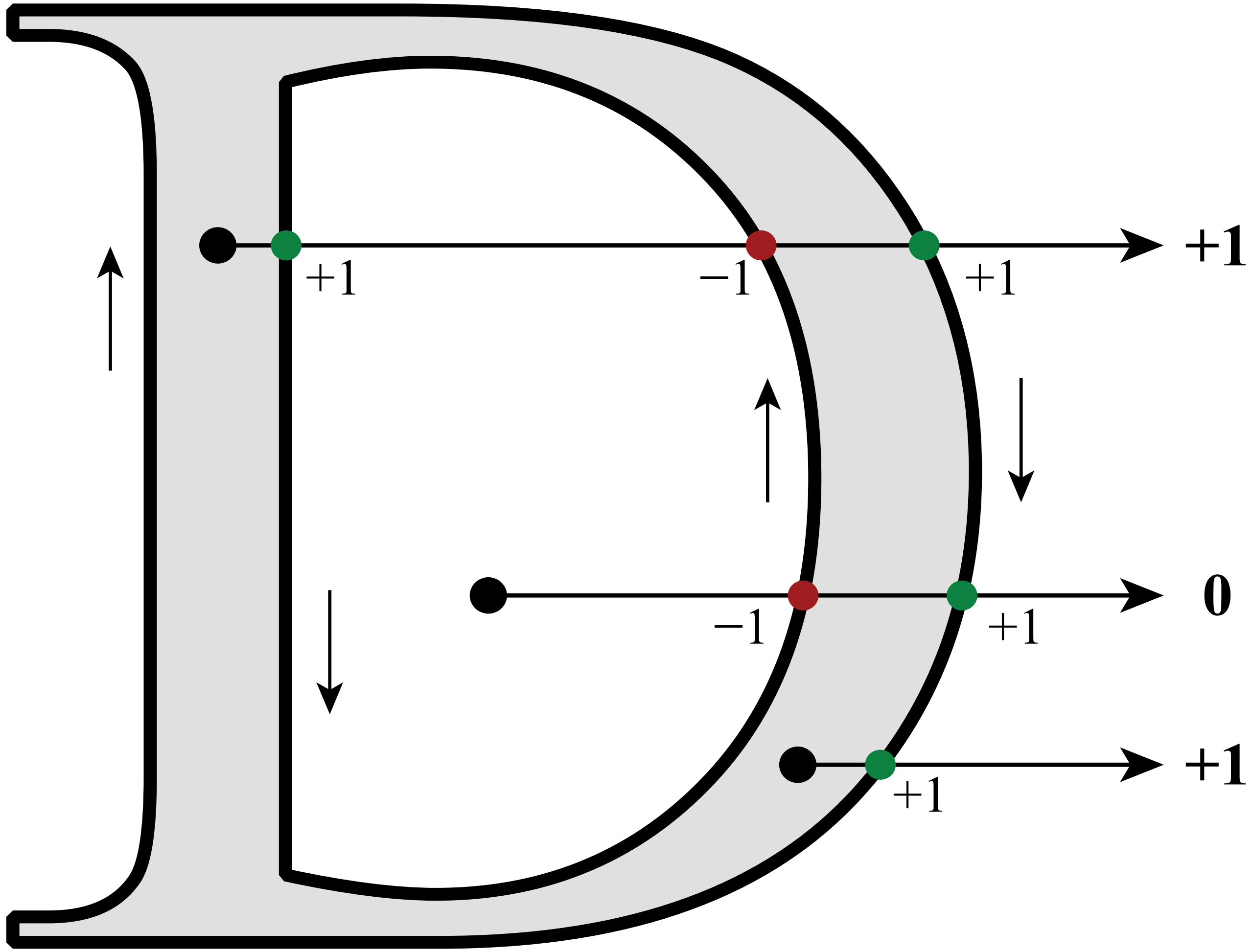
Winding Number

- To calculate winding number, fire a ray from point being rendered to infinity
- Direction doesn’t matter, so use +*x* direction for convenience
- Look for contour intersections along the ray

Winding Number

- When contour crosses ray from left to right, increment winding number
- When contour crosses ray from right to left, decrement winding number
- Or other way around, as long as consistent

GPU-Centered Font Rendering Directly from Glyph Outlines



Quadratic Bézier Curve

- Three control points $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$

- Parametric curve with $0 \le t \le 1$:

$$\mathbf{C}(t) = (1-t)^2\,\mathbf{p}_1 + 2t(1-t)\,\mathbf{p}_2 + t^2\mathbf{p}_3$$

Ray-Curve Intersection

- Translate control points so ray origin is (0,0)
- Assume ray direction is +*x* axis
- Solve for values of *t* where *y* coordinate of Bézier curve is zero

Ray-Curve Intersection

- Let $\mathbf{p}_i = (x_i, y_i)$
- Ray intersects curve at roots of polynomial

$$(y_1 - 2y_2 + y_3)t^2 - 2(y_1 - y_2)t + y_1$$

$$a = y_1 - 2y_2 + y_3 \qquad b = y_1 - y_2 \qquad c = y_1$$

Ray-Curve Intersection

- Roots t_1 and t_2 given by

$$t_1 = \frac{b - \sqrt{b^2 - ac}}{a} \qquad t_2 = \frac{b + \sqrt{b^2 - ac}}{a}$$

- If *a* near zero, use root of linear polynomial:

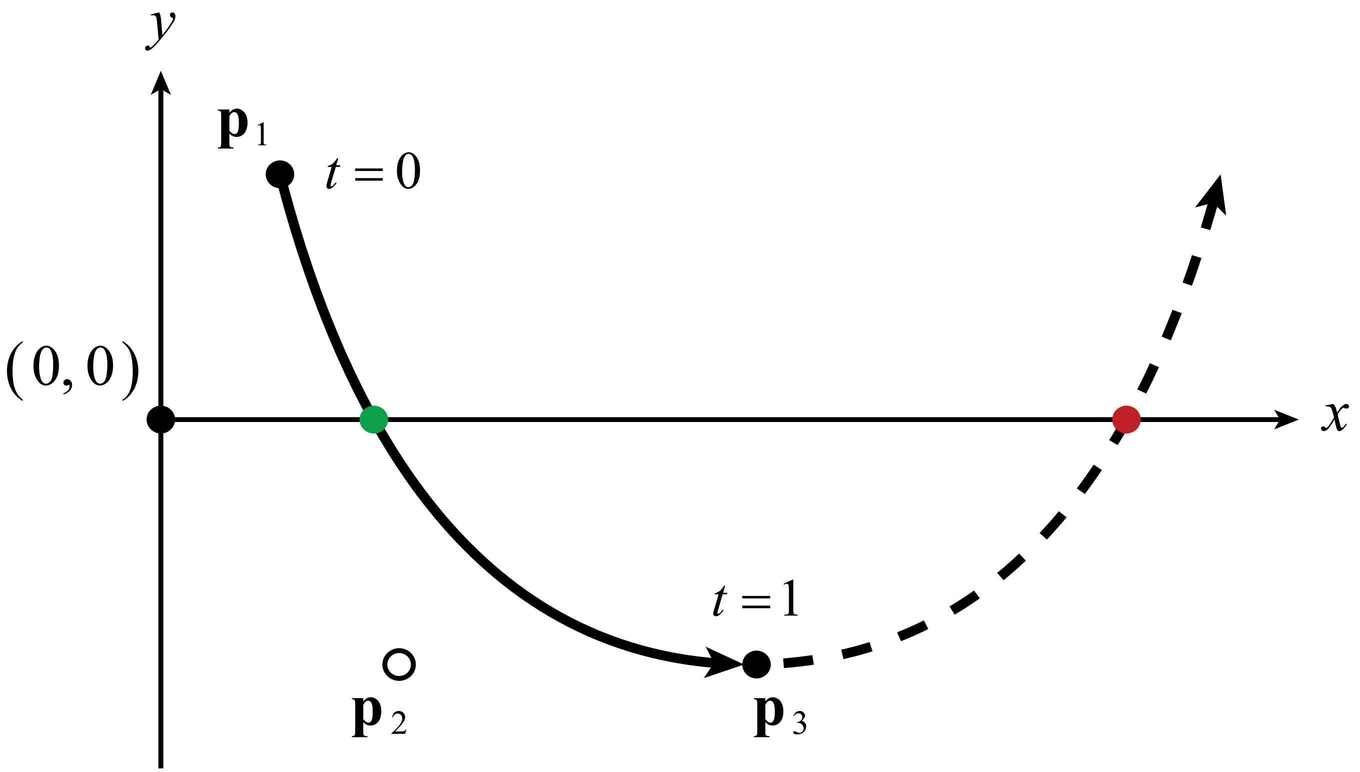
$$t_1 = t_2 = \frac{c}{2b}$$

Ray-Curve Intersection

- Valid intersection at t_i when:
  - $0 \le t_i \le 1$ (between curve endpoints)
  - $C_x(t_i) \ge 0$ (at positive distance along ray)

- $t_i = 1$ specifically disallowed
  - Corresponds to intersection at $t_i = 0$ on next Bézier curve, and don't want to count twice

Ray-Curve Intersection



Ray-Curve Intersection

- Increment or decrement winding number?
- Look at *y* values in range $0 \le t_i \le 1$
  - Positive before t_i or negative after t_i: increment
  - Negative before t_i or positive after t_i: decrement
- Can’t rely on derivative
  - Zero if ray tangent to curve

Robustness

- Sound from purely mathematical standpoint
- But plagued by numerical precision errors!
- Floating-point limits cause huge problems for roots near endpoints where $t_i = 0$ or $t_i = 1$

Numerical Precision Errors

- Produce sparkle and streak artifacts
- Hacks like epsilons and coordinate perturbation just shift problem cases around
- Need something that’s 100% robust

Slug Algorithm

- Calculates winding number
  - Input is arbitrary set of closed contours composed of quadratic Bézier curves
- Performs antialiasing
  - Determines fractional coverage at each pixel

Priority #1: Works Correctly

- Robust for all valid inputs
  - Meaning any floating-point coordinates that are not infinity or NaN
- No distortion of glyph outlines
- No sparkle artifacts

Equivalence Class Algorithm

- Infinite problem space reduced to a finite number of equivalence classes
- Same procedure followed for all cases in each equivalence class
- Same abstract idea as Marching Cubes

Bézier Curve Classification

- Look at *y* coordinates of the three control points
- Each positive, negative, or zero
- 27 classes based on these states

Bézier Curve Classification

- It turns out we can do better than 27 classes
- Classify each control point based on whether *y* coordinate is nonnegative or negative
- Only 8 equivalence classes

Bézier Curve Classification

- Roots (ray intersections) always occur in same way for all cases in each class
- We care about places where curve transitions between nonnegative and negative
- Only have to decide how to modify winding number for each root

Winding Number Modification

- Consider derivative of *y* coordinate:

$$y'(t) = 2at - 2b$$

$$a = y_1 - 2y_2 + y_3 \qquad b = y_1 - y_2$$

Winding Number Modification

- An observation about the roots for nonzero discriminant *D*:

$$t_1 = \frac{b - \sqrt{D}}{a} \qquad t_2 = \frac{b + \sqrt{D}}{a}$$

$$D = b^2 - ac$$

$$y'(t_1) = -2\sqrt{D} \qquad y'(t_2) = +2\sqrt{D}$$

Winding Number Modification

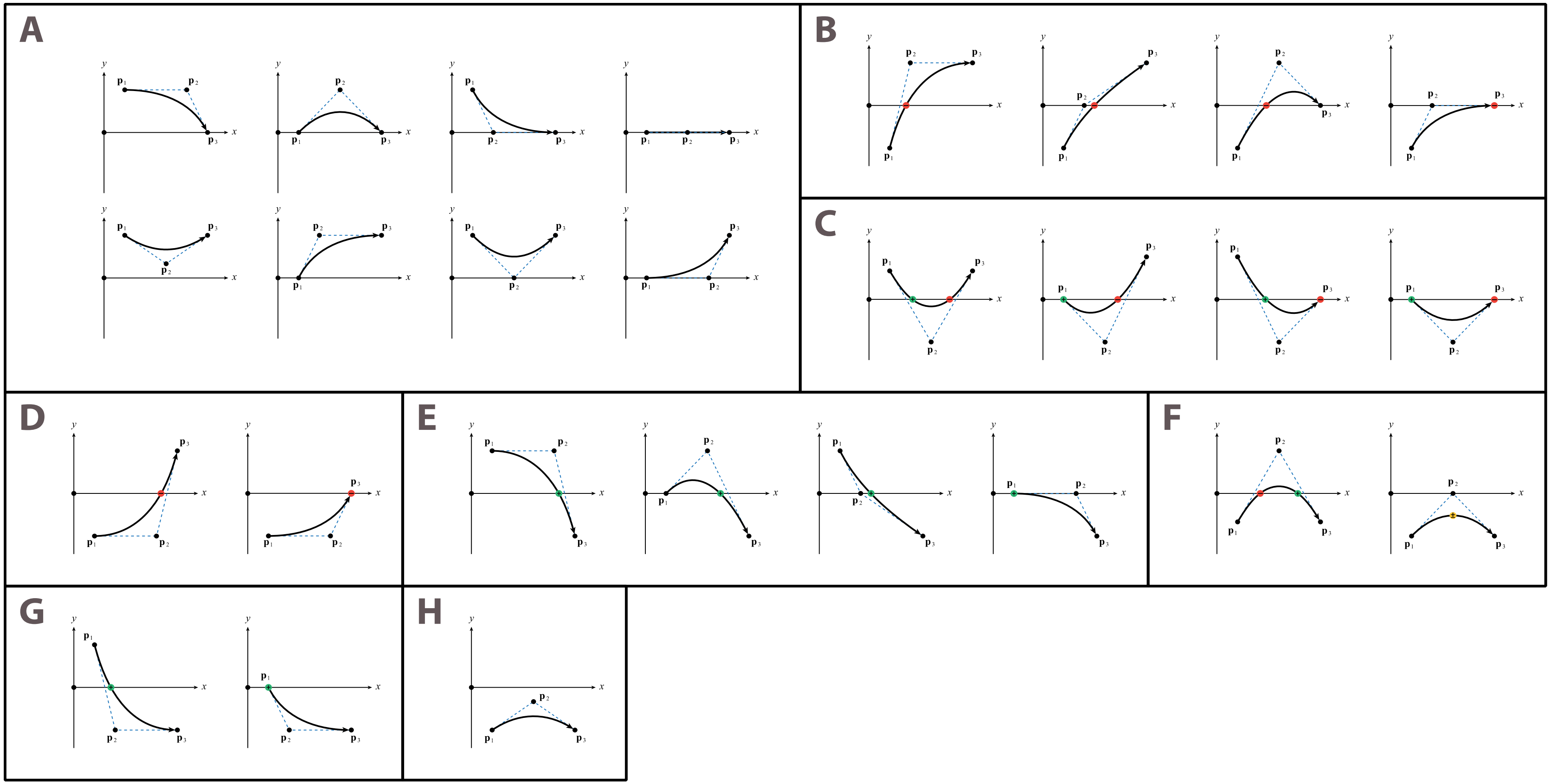
- Root at t_1 always crosses ray from left to right
  - Going from nonnegative to negative
  - Always increment winding number

- Root at t_2 always crosses ray from right to left
  - Going from negative to nonnegative
  - Always decrement winding number

Winding Number Modification

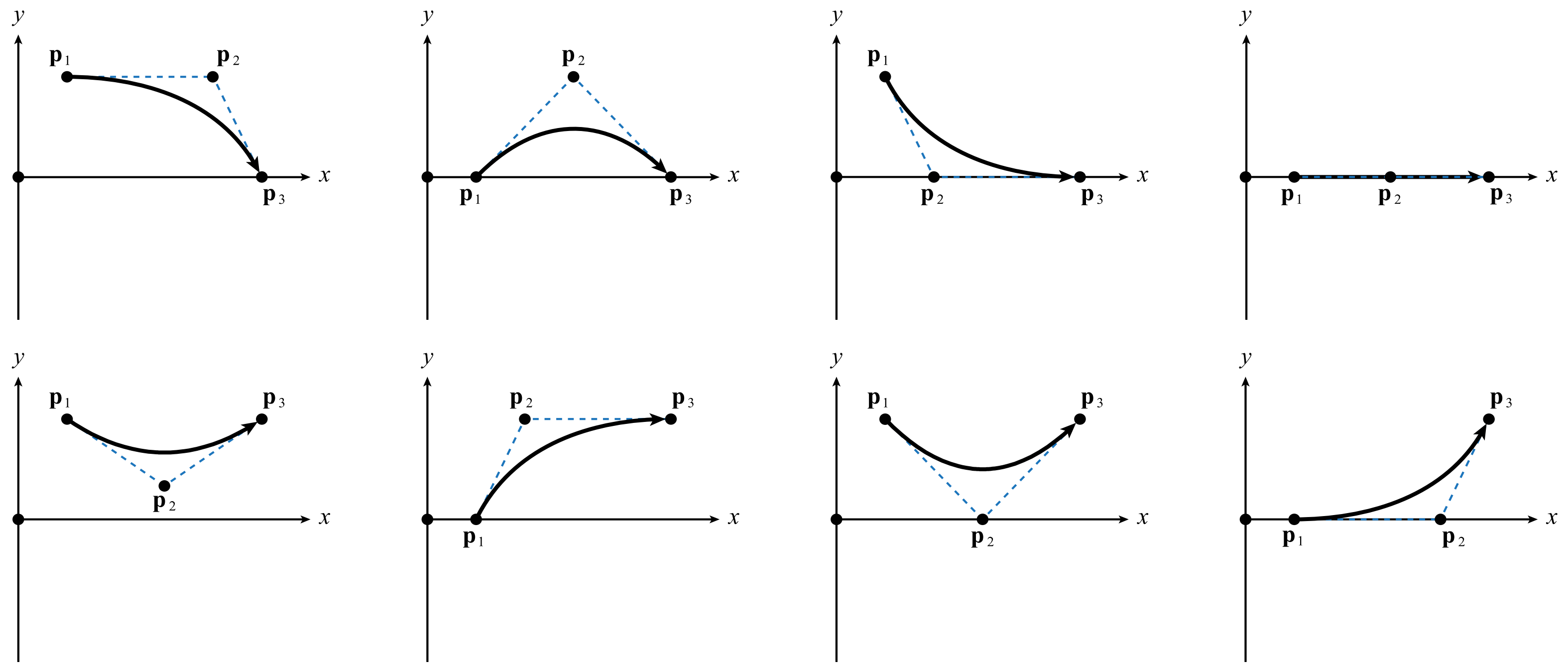
- We can also incorporate cases where ray intersects an endpoint tangentially
- Winding number modified only when transition between nonnegative and negative occurs
- x coordinate at transition must be positive

GPU-Centered Font Rendering Directly from Glyph Outlines



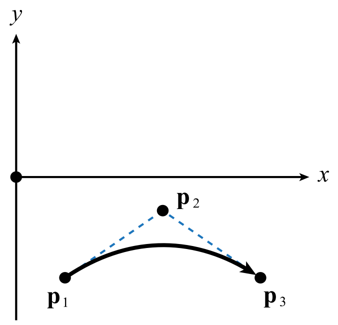
Class A: All Nonnegative

- Nothing happens to winding number



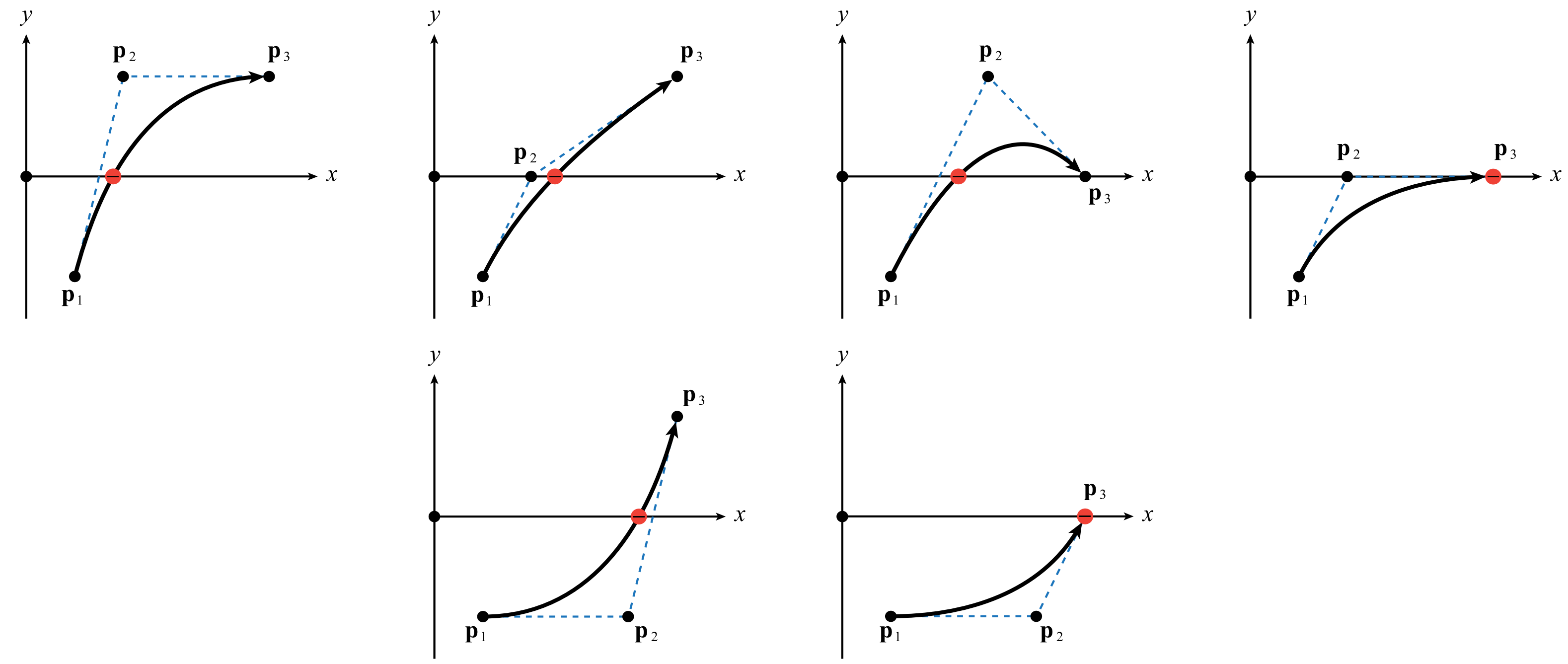
Class H: All Negative

- Nothing happens to winding number



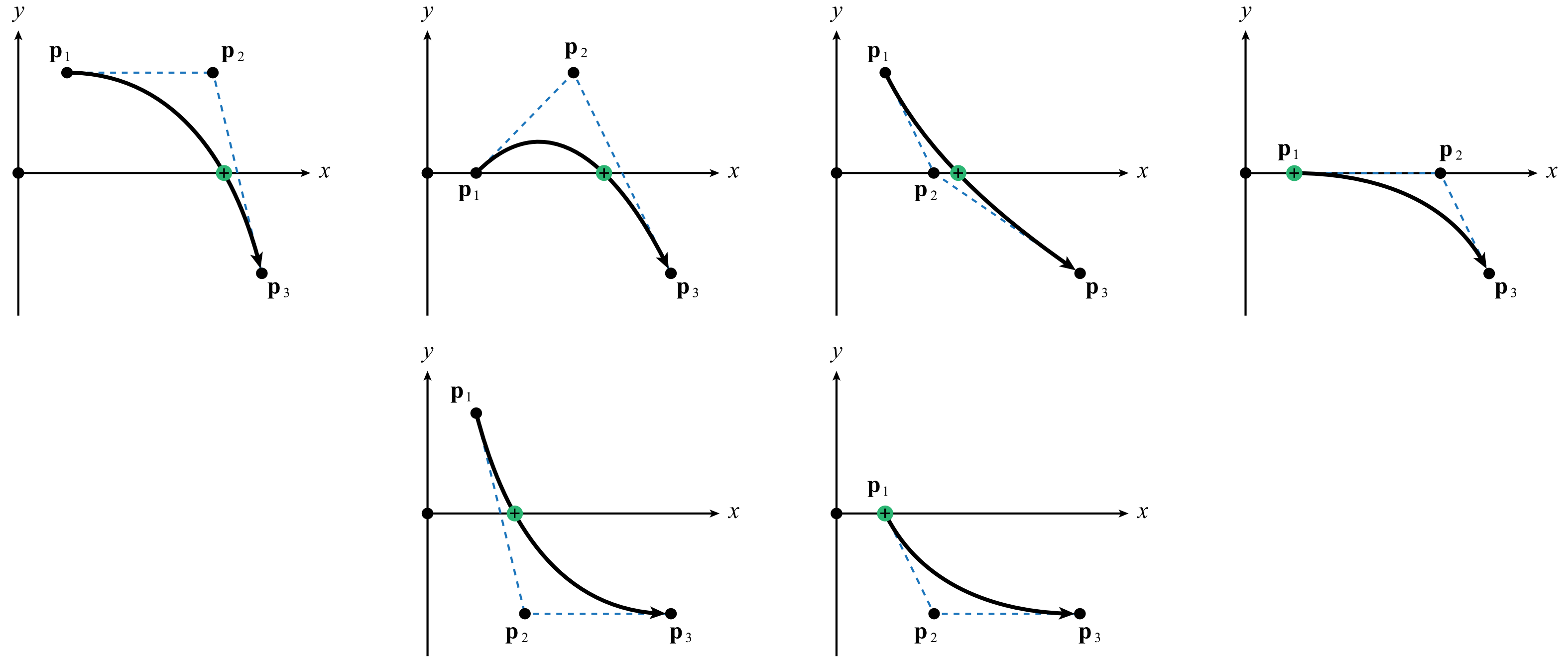
Classes B and D: One Transition

- Winding number decremented if $x(t_2) > 0$



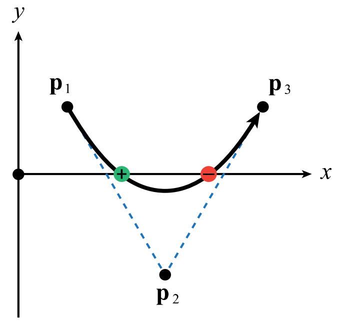
Classes E and G: One Transition

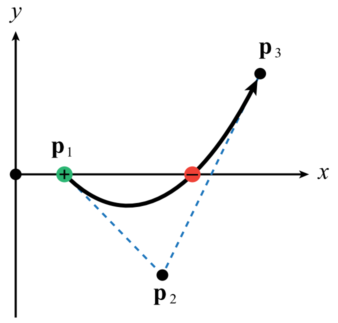
- Winding number incremented if $x(t_1) > 0$

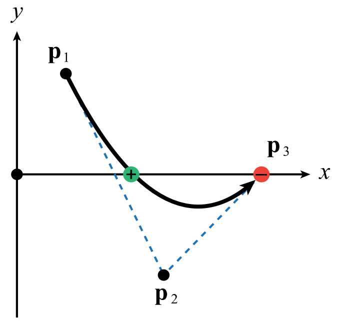


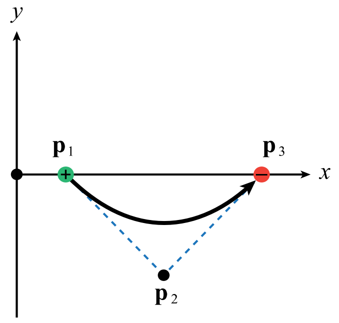
Class C: Two Transitions

- Winding number incremented if $x(t_1) > 0$
- Winding number decremented if $x(t_2) > 0$



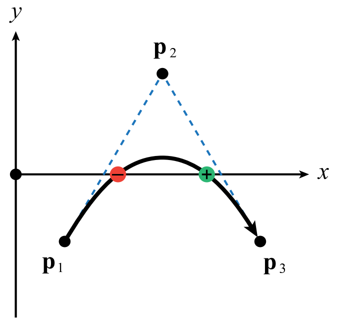


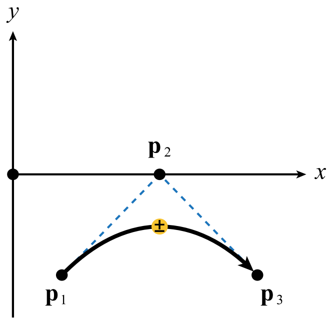




Class F: Two Transitions

- Winding number incremented if $x(t_1) > 0$
- Winding number decremented if $x(t_2) > 0$





Discriminant Clamping

- In classes C and F, we could have a negative discriminant *D*
- To handle with uniformity, clamp *D* to zero
- Always two transitions at same *x* coordinate, so guaranteed to cancel each other out

Root Calculation

```
float2 SolvePoly(float4 p12, float2 p3)
{
    float2 a = p12.xy - p12.zw * 2.0 + p3;    // Calculate coefficients.
    float2 b = p12.xy - p12.zw;
    float ra = 1.0 / a.y;
    float rb = 0.5 / b.y;

    float d = sqrt(max(b.y * b.y - a.y * p12.y, 0.0));   // Clamp discriminant to zero.
    float t1 = (b.y - d) * ra;
    float t2 = (b.y + d) * ra;

    if (abs(a.y) < epsilon) t1 = t2 = p12.y * rb;    // Handle linear case where |a| ≈ 0.

    // Return x coordinates at t1 and t2.
    return (float2((a.x * t1 - b.x * 2.0) * t1 + p12.x,
                   (a.x * t2 - b.x * 2.0) * t2 + p12.x));
}
```

Root Eligibility

- We know what to do for each root
- Just need to decide whether to actually do it!
- Use a lookup table for root eligibility

Root Eligibility

- Nonnegative/negative gives us 3-bit state
  - Just use sign bits of y coordinates
- Look up 2-bit root eligibilities for t_1 and t_2
- Total LUT size is a tiny 16 bits

Root Eligibility Lookup Table

Class	$y_3 < 0$	$y_2 < 0$	$y_1 < 0$	Root 2	Root 1
A	0	0	0	0	0
B	0	0	1	1	0
C	0	1	0	1	1
D	0	1	1	1	0
E	1	0	0	0	1
F	1	0	1	1	1
G	1	1	0	0	1
H	1	1	1	0	0
				0x2E	0x74

Calculating Root Codes

```
uint CalcRootCode(float y1, float y2, float y3)
{
    uint i1 = asuint(y1) >> 31U;
    uint i2 = asuint(y2) >> 30U;
    uint i3 = asuint(y3) >> 29U;

    uint shift = (i2 & 2U) | (i1 & ~2U);
    shift = (i3 & 4U) | (shift & ~4U);

    return ((0x2E74U >> shift) & 0x0101U);
}
```

```
bool TestCurve(uint code)
{
    return (code != 0U);
}

bool TestRoot1(uint code)
{
    return ((code & 1U) != 0U);
}

bool TestRoot2(uint code)
{
    return (code > 1U);
}
```

Ternary Logic Instruction

- Recent Nvidia GPUs have a ternary logic instruction (LOP3)
- Maps arbitrary 3-bit input to 1-bit output with 8-bit lookup table encoded in the instruction
- Perfect fit for our algorithm!

Ternary Logic Instruction

- Instruction not directly accessible from pixel shader code
- Compiler can recognize arbitrary sequence of AND, OR, XOR, NOT operations and generate single ternary instruction

Ternary Logic Instruction

- With LOP3, root code calculation requires only 2 instructions where otherwise need at least 7

- Shader gets about 4% faster in all cases

Root Codes with Ternary Logic

```
int2 CalcRootCode(float y1, float y2, float y3)
{
    int a = asint(y1);
    int b = asint(y2);
    int c = asint(y3);

    return (int2(~a & (b | c) | (~b & c),
                 a & (~b | ~c) | (b & ~c)));
}
```

```
bool TestCurve(int2 code)
{
    return ((code.x | code.y) < 0);
}

bool TestRoot1(int2 code)
{
    return (code.x < 0);
}

bool TestRoot2(int2 code)
{
    return (code.y < 0);
}
```

Total Winding Number

```
int winding = 0;
for (all Bézier curves)
{
    float4 p12 = first (.xy) and second (.zw) control points
    float2 p3 = third (.xy) control point

    code = CalcRootCode(p12.y, p12.w, p3.y);
    if (TestCurve(code))
    {
        float2 r = SolvePoly(p12, p3);

        if ((TestRoot1(code)) && (r.x > 0.0)) winding += 1;
        if ((TestRoot2(code)) && (r.y > 0.0)) winding -= 1;
    }
}

if (winding != 0) then pixel is inside glyph outline
```

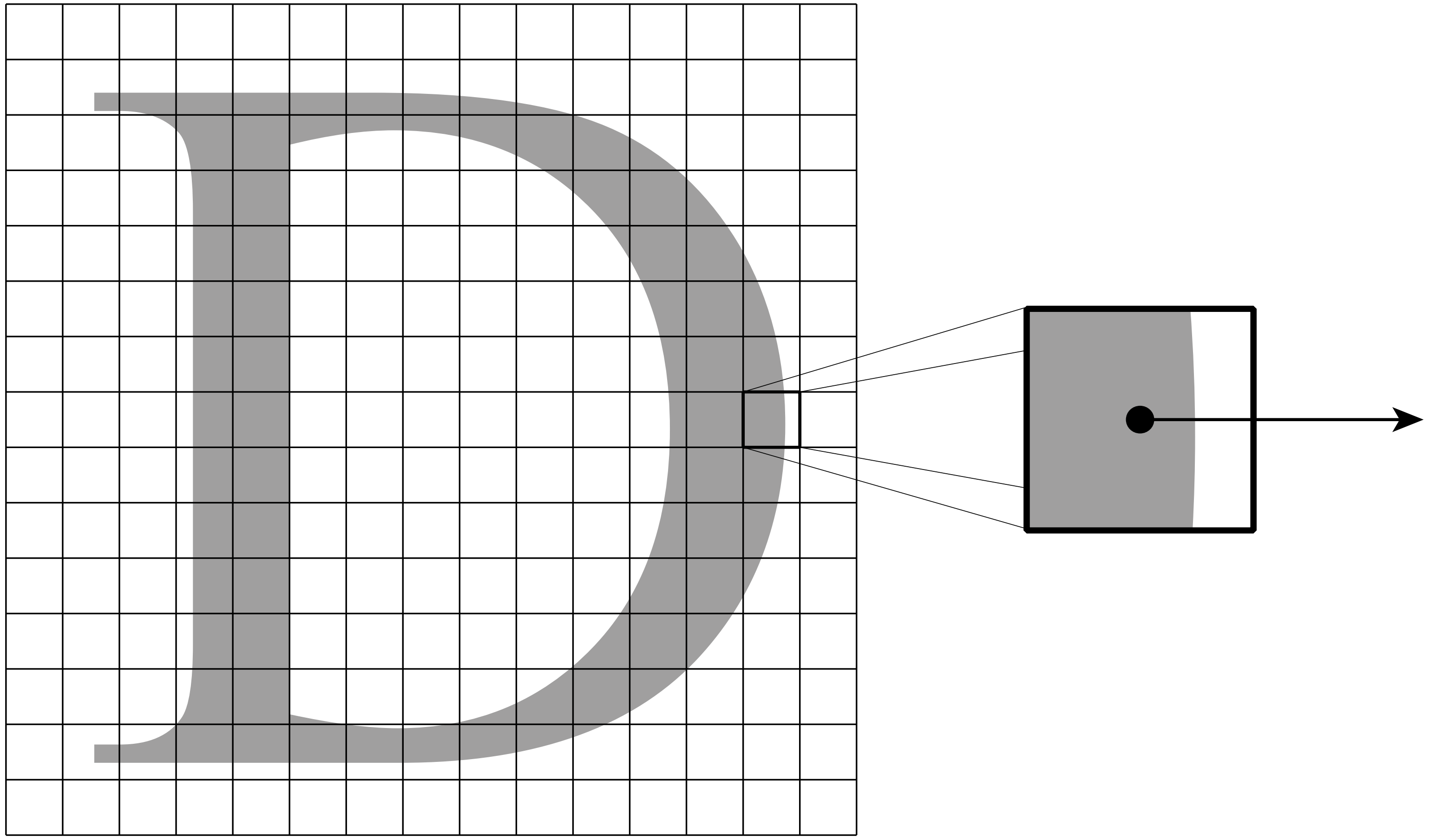
Priority #2: Looks Good

- Performs accurate antialiasing
- Handles arbitrary transforms well
- Handles minification well

Fractional Coverage

- Integer winding number produces simple in/out state for each pixel
- Correct, but has jagged edges everywhere
- We need fractional pixel coverage values

GPU-Centered Font Rendering Directly from Glyph Outlines



Fractional Coverage

- Ray origin is at pixel center
- Previously, we incremented or decremented winding number when $x(t_i) > 0$
- Now, we add or subtract the fractional distance ray makes it through pixel before intersection

Fractional Coverage

- Let *u* be number of pixels per em
  - Scales coordinates so that width of pixel = 1 unit

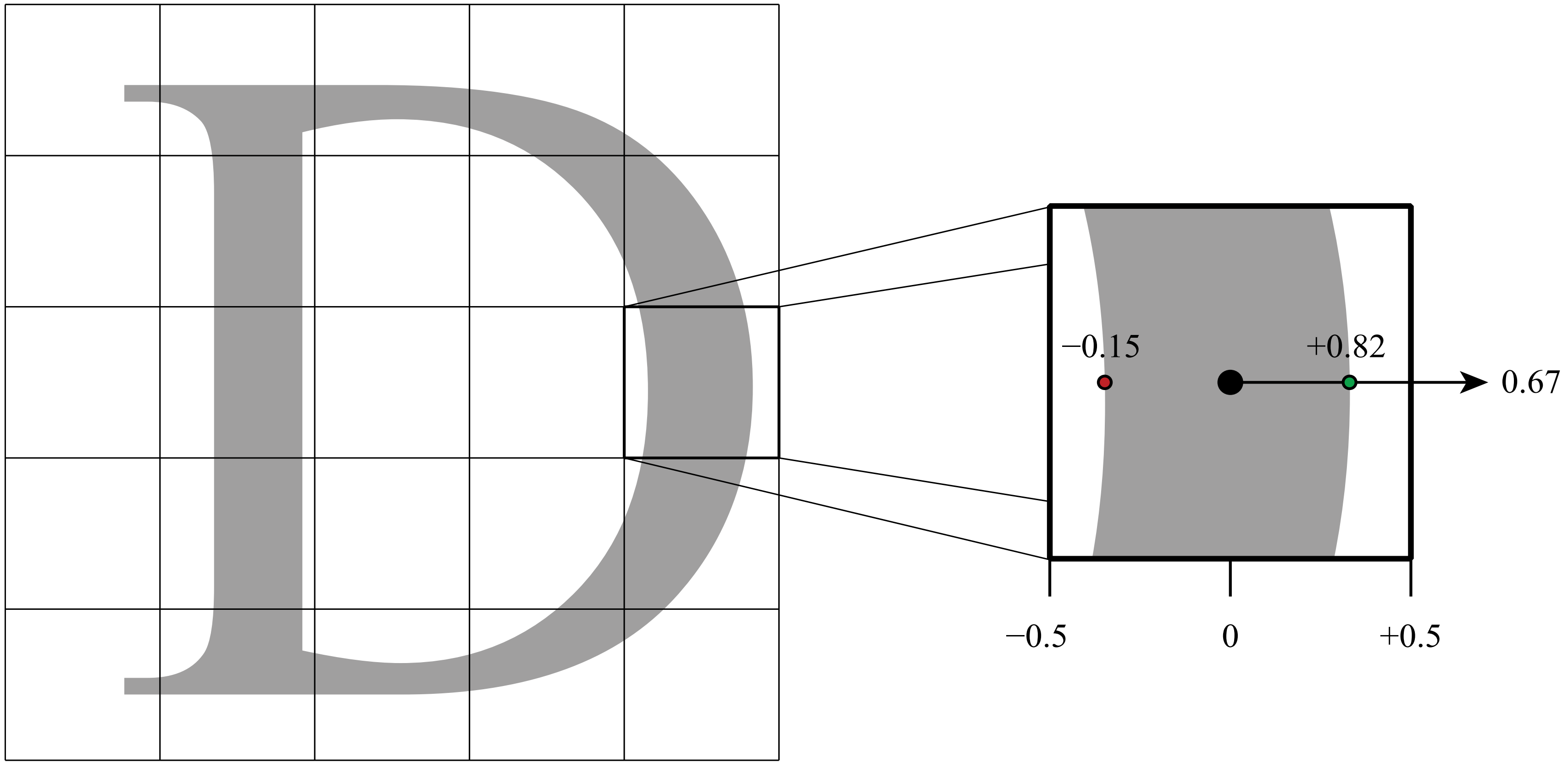
- Always change winding number (WN) by

$$\operatorname{saturate}\left(u \cdot x\left(t_i\right) + \tfrac{1}{2}\right)$$

Fractional Coverage

- If $u \cdot x(t_i) \leq -0.5$, then no change to WN
- If $u \cdot x(t_i) \geq 0.5$, then WN always changed by 1
- In between, WN changed by fractional value
- Accounts for multiple curves per pixel

GPU-Centered Font Rendering Directly from Glyph Outlines



Fractional Winding Number

```
float coverage = 0.0;
for (all Bézier curves)
{
    float4 p12 = first (.xy) and second (.zw) control points
    float2 p3 = third (.xy) control point

    code = CalcRootCode(p12.y, p12.w, p3.y);
    if (TestCurve(code))
    {
        float2 r = SolvePoly(p12, p3) * pixelsPerEm;

        if (TestRoot1(code)) coverage += saturate(r.x + 0.5);
        if (TestRoot2(code)) coverage -= saturate(r.y + 0.5);
    }
}
```

Antialiasing

- This gives us excellent *1D* antialiasing
  - Looks great when curves are mostly vertical
- Doesn’t work well for mostly horizontal curves
- So fire a ray in the *y* direction, too
  - Looks great when curves are mostly horizontal

Antialiasing

- Calculate coverage for two rays at each pixel
  - One in *x* direction, and one in *y* direction
  - Note pixels per em could be different in *x* and *y*
- Combine two coverage values for good 2D antialiasing
  - Lots of ways to calculate weighted average

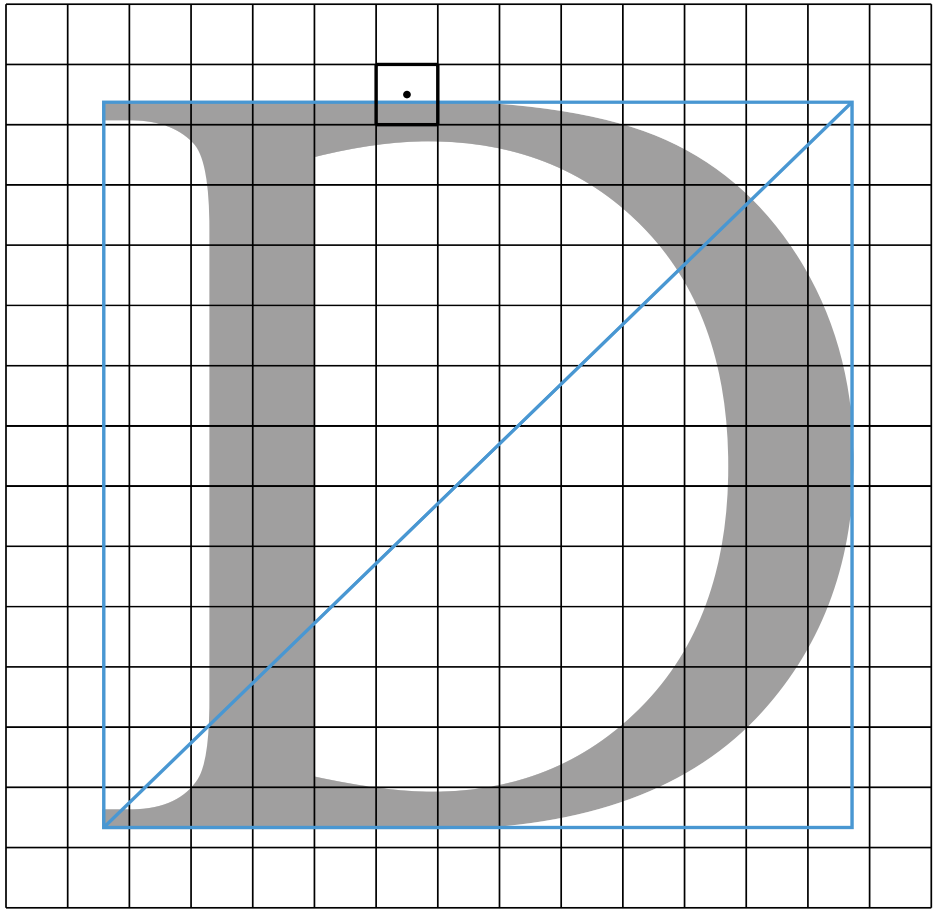
Antialiasing

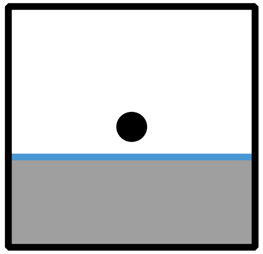
- Output is linear coverage value
- Works best when blended into sRGB framebuffer

Bounding Box Dilation

- Draw one quad per glyph coinciding with glyph’s bounding box
- GPU fills pixels with centers covered by quad
- Could miss pixels on boundary with up to 50% fractional coverage value

GPU-Centered Font Rendering Directly from Glyph Outlines





Bounding Box Dilation

- Must dilate bounding box by half pixel width
- In em space, dilate by 0.5 / font size
- If size dynamically change, need to estimate smallest on-screen pixels per em

Minification

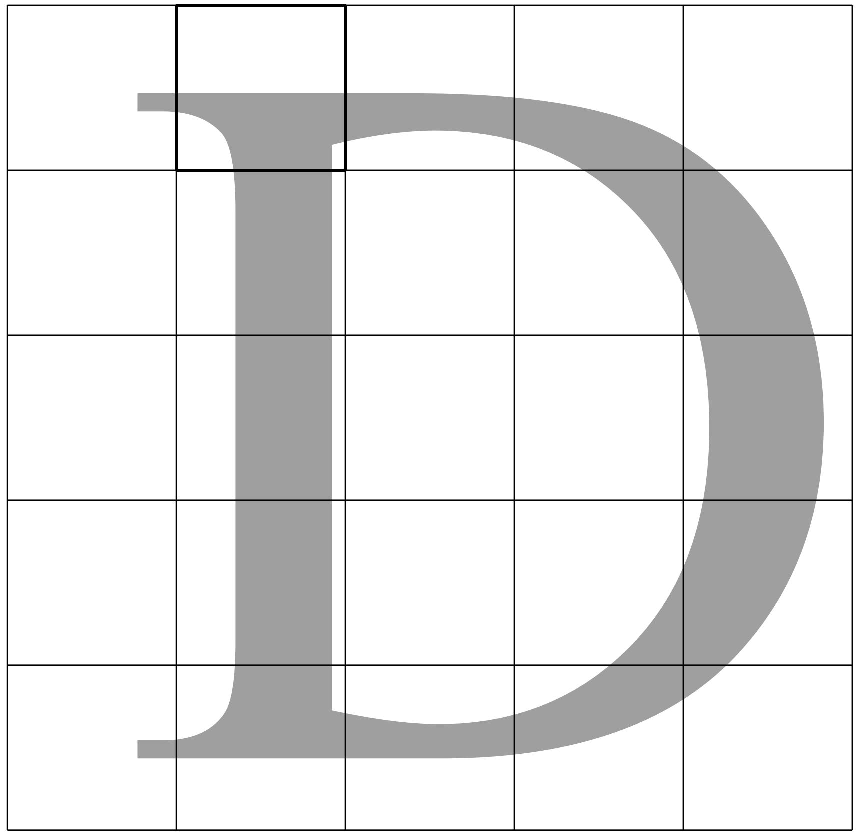
- At very small font sizes, lots of detail can occur inside each pixel
- Can’t be captured by single sample position at pixel center

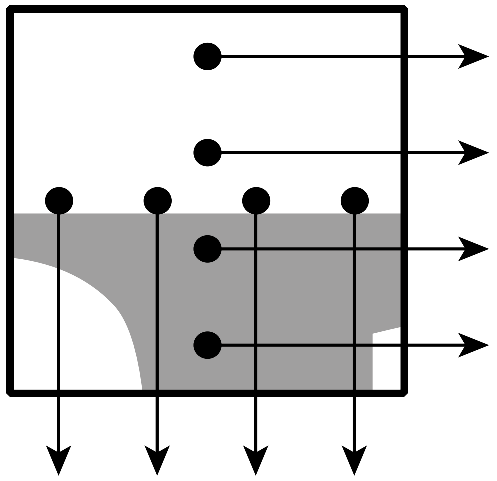
Adaptive Supersampling

- As pixels get larger in em space, increase number of samples

- Use screen space derivatives to dynamically calculate sample counts for horizontal and vertical rays

GPU-Centered Font Rendering Directly from Glyph Outlines





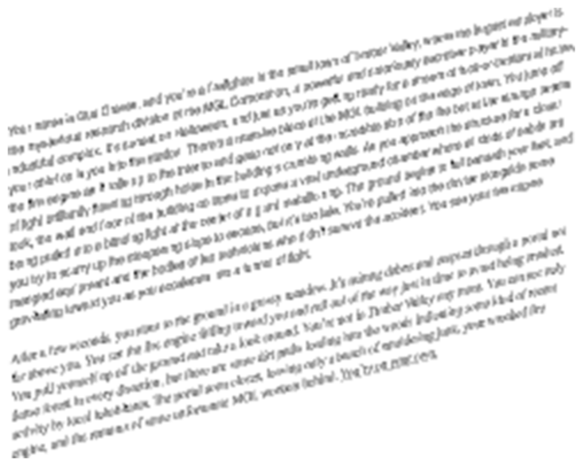
Adaptive Supersampling

- Example sample count calculation
    - 1–4 samples per pixel in each direction

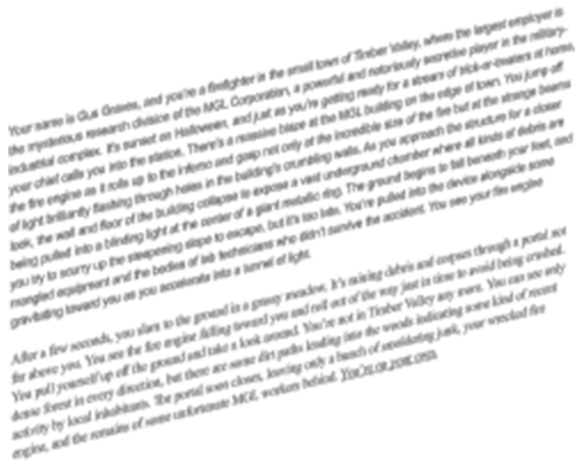
```
float2 emsPerPixel = fwidth(renderCoord);
int2 sampleCount = clamp(int2(emsPerPixel * 32.0 + 1.0), int2(1, 1), int2(4, 4));
```

Adaptive Supersampling

Single sample



Supersampling



Priority #3: Runs Fast

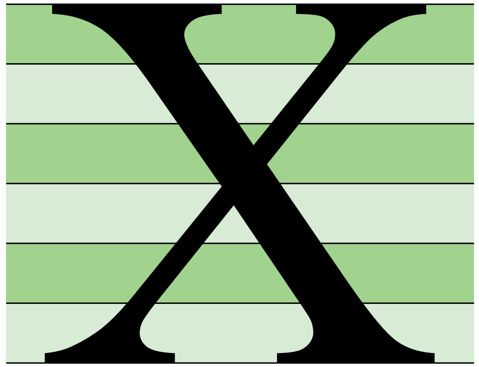
- Minimize raw computation
  - We want to examine as few Bézier curves as possible in the pixel shader
- Promote high GPU resource utilization
  - We want low thread divergence in the pixel shader

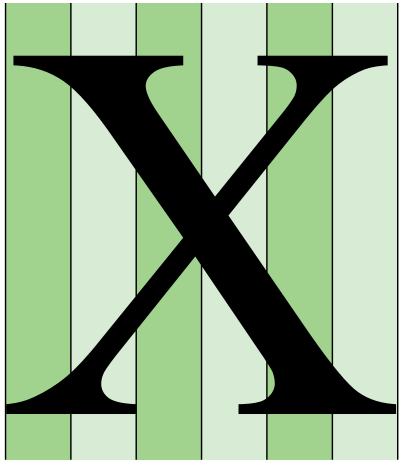
Computation

- Looking for ray intersections with all Bézier curves would be very slow
- Many curves far away from ray and never contribute to coverage (classes A and H)
- Need to reduce active set of curves

Banding

- Divide glyph's bounding box into many horizontal and vertical bands





Banding

- Bézier curves are sorted into the bands
    - A curve can belong to multiple bands
    - When rendering, band selected based on ray origin
- Doesn’t matter how large pixel footprint gets
    - Pixel size only matters in ray direction
    - Band parallel to ray extends forever

Banding

- Perfectly horizontal lines are never added to horizontal bands
- Perfectly vertical lines are never added to vertical bands
- Ray intersections with these can’t happen

Banding

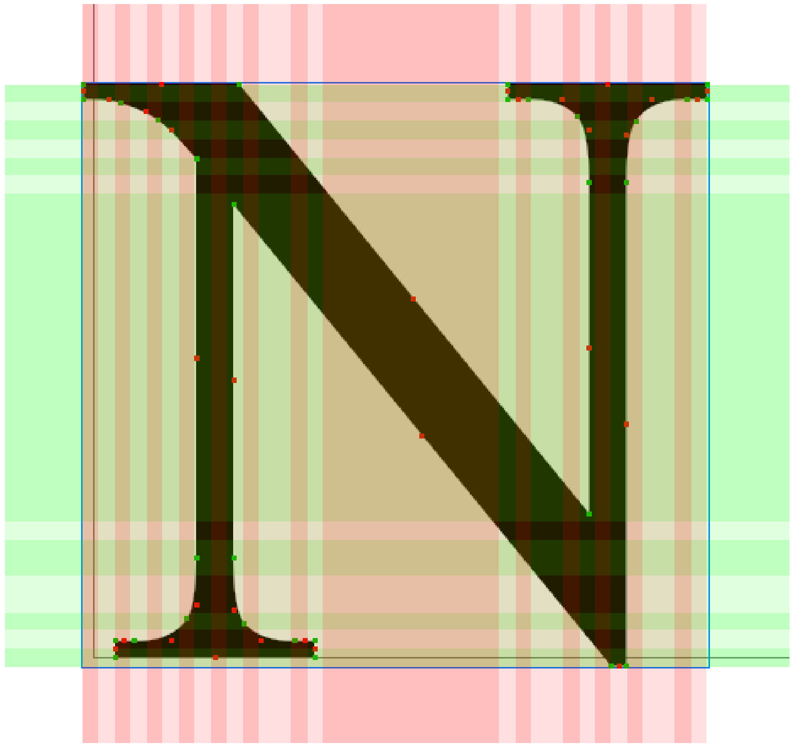
- Further dividing into cells causes problems
  - Pixel could cover multiple cells along ray direction
  - Those cells often won't have disjoint curve sets
  - Can't calculate final winding number without additional per-cell fix-ups that aren't robust
- Bands are a much cleaner and faster solution

Banding

- Using large numbers of bands is faster
  - Allows fewer curves per band
- Minimize number of curves in worst band
  - GPU thread coherence makes shader wait for highest number of loop iterations in a group of pixels (32 or 64, hardware dependent)

Banding

- Can merge data for bands containing identical sets of Bézier curves



Curve Sorting

- Curves in each horizontal band are sorted in descending order by the maximum *x* coordinate of the three control points
- This is an early-exit optimization that makes the shader about twice as fast compared to not sorting curves at all

Curve Sorting

- Translate control points so that pixel center is at (0,0), and perform test:

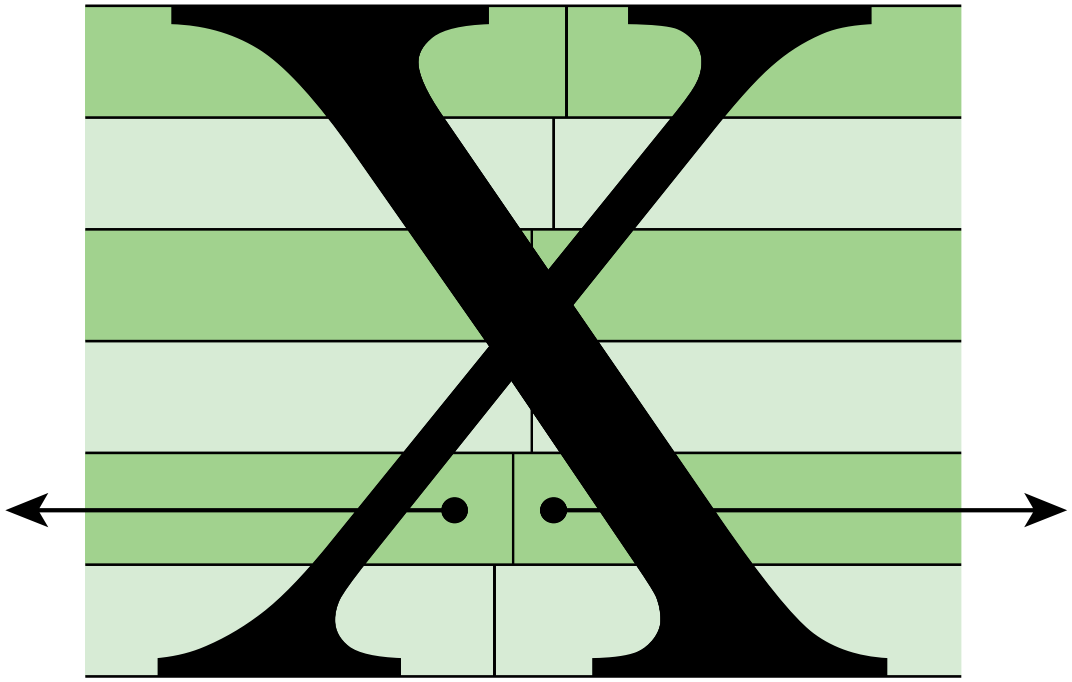
```
if (max(max(p12.x, p12.z), p3.x) * pixelsPerEm.x < -0.5) break;
```

- If true, then it’s not possible to hit this curve or any that follow
  - Sorted in descending order, so must also be true for all later curves in the band

Symmetric Band Optimization

- Can do even better at large font sizes
- Also sort in ascending order by minimum *x* coordinate of the three control points
- Use a left-pointing ray when pixel *x* coordinate is less than a band split value

Symmetric Band Optimization



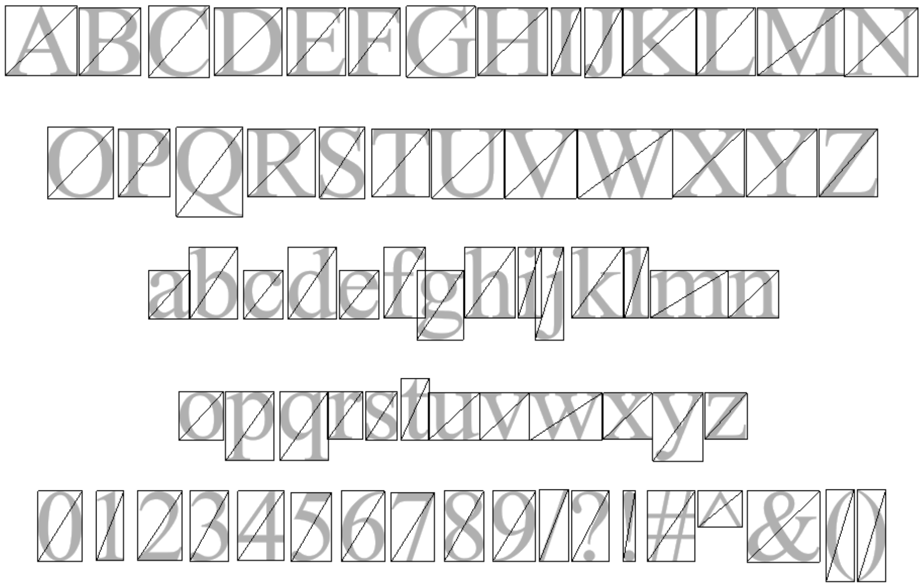
Symmetric Band Optimization

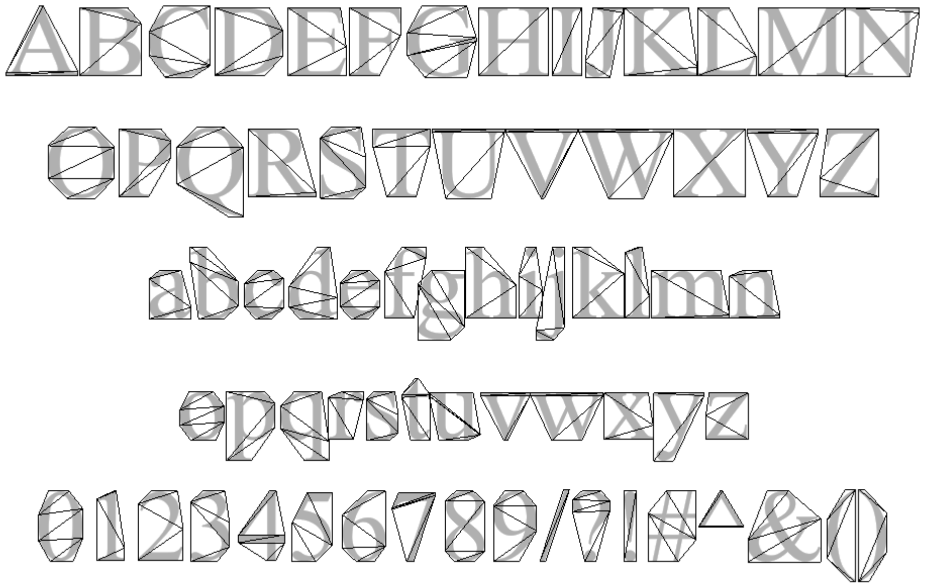
- Sorting and split values also apply to vertical bands
- Splits values introduce divergence in shader
  - Faster for large font sizes where lots of pixels will choose same execution path
  - Slower for small font sizes due to decoherence

Bounding Polygons

- Glyphs tend to have empty space near the corners of their bounding boxes
- Clip these corners off to reduce pixels filled
- Adds more triangles, but roughly 10% faster with larger font sizes

Bounding Polygons





Bounding Polygons

- As with symmetric band splits, bounding polygons increase performance for large font sizes, but can hurt at small font sizes
- Greater number of triangles increase number of pixels double-shaded in 2x2 quads along triangle edges

Rectangle Primitives

- Even when rendering quads, double-shading along the interior edge can be a significant expense at small font sizes

- Expense can be eliminated for text that's aligned to screen axes

Rectangle Primitives

- Most GPUs support rectangle primitives
  - Exposed through `GL_NV_fill_rectangle` extension
- Specify three vertices, and screen-aligned enclosing rect is drawn without internal edge
- Up to 15% faster for typical font sizes

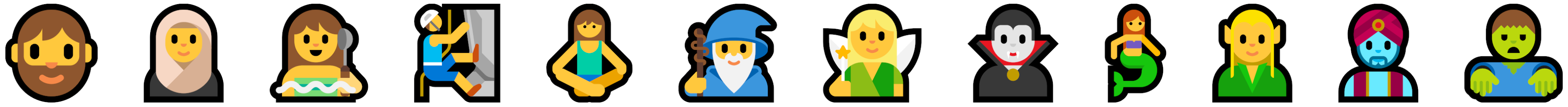
Rectangle Primitives

- Can be combined with conservative rasterization to handle glyph dilation

- Automatically shades pixels with any amount of coverage

Multicolor Glyphs

- Microsoft emoji font uses vector artwork
  - Based on same TrueType quadratic Bézier curves
- Multiple layers composited back to front
  - Adds an outer loop to the pixel shader



Data Stored in Two Texture Maps

- Curve texture
  - 4-channel 16-bit floating-point
  - Stores all Bézier curve control points

- Band texture
  - 4-channel 16-bit integer
  - Stores lists of curves for all bands

Curve Texture

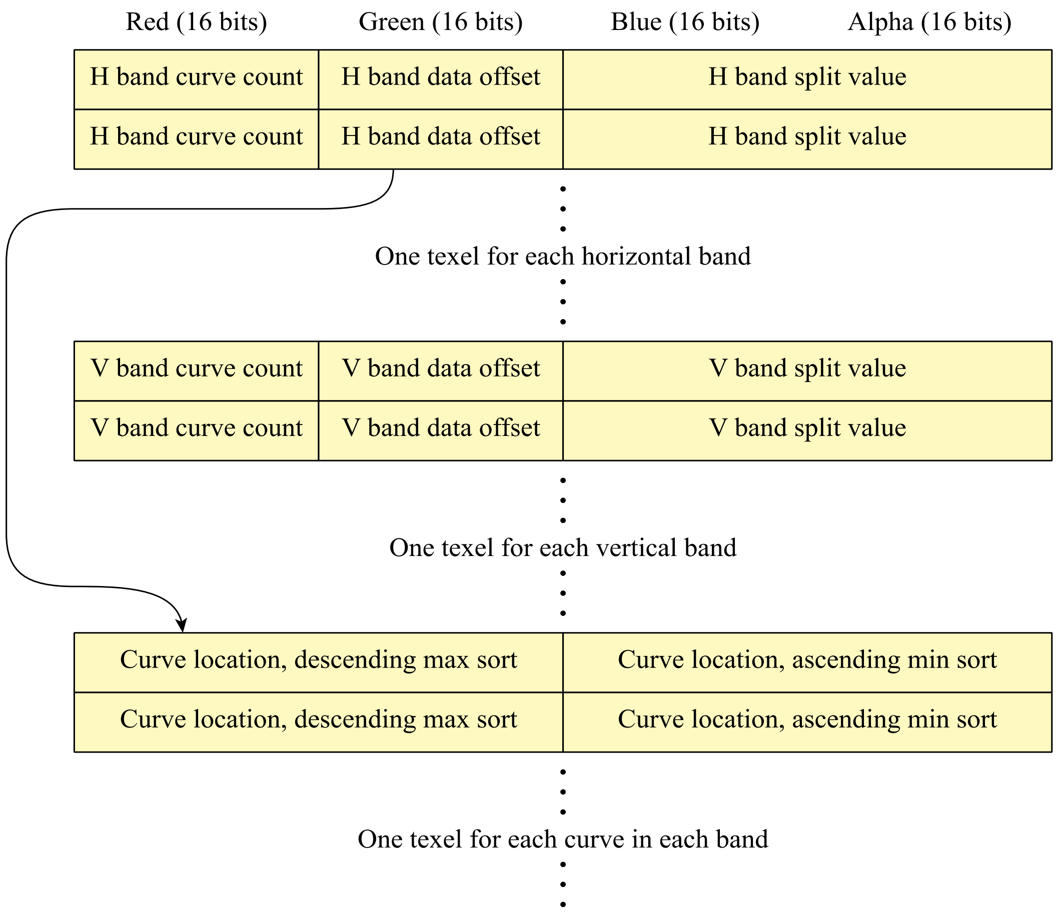
- Third control point of one curve is always same as first control point of next curve in contour

Red (16 bits)	Green (16 bits)	Blue (16 bits)	Alpha (16 bits)
p1.x (curve 1)	p1.y (curve 1)	p2.x (curve 1)	p2.y (curve 1)
p3.x (curve 1) p1.x (curve 2)	p3.y (curve 1) p1.y (curve 2)	p2.x (curve 2)	p2.y (curve 2)
p3.x (curve 2) p1.x (curve 3)	p3.y (curve 2) p1.y (curve 3)	p2.x (curve 3)	p2.y (curve 3)

⋮

Band Texture

- Each glyph has list of H bands and V bands
- Each band contains list of curves, sorted in both directions

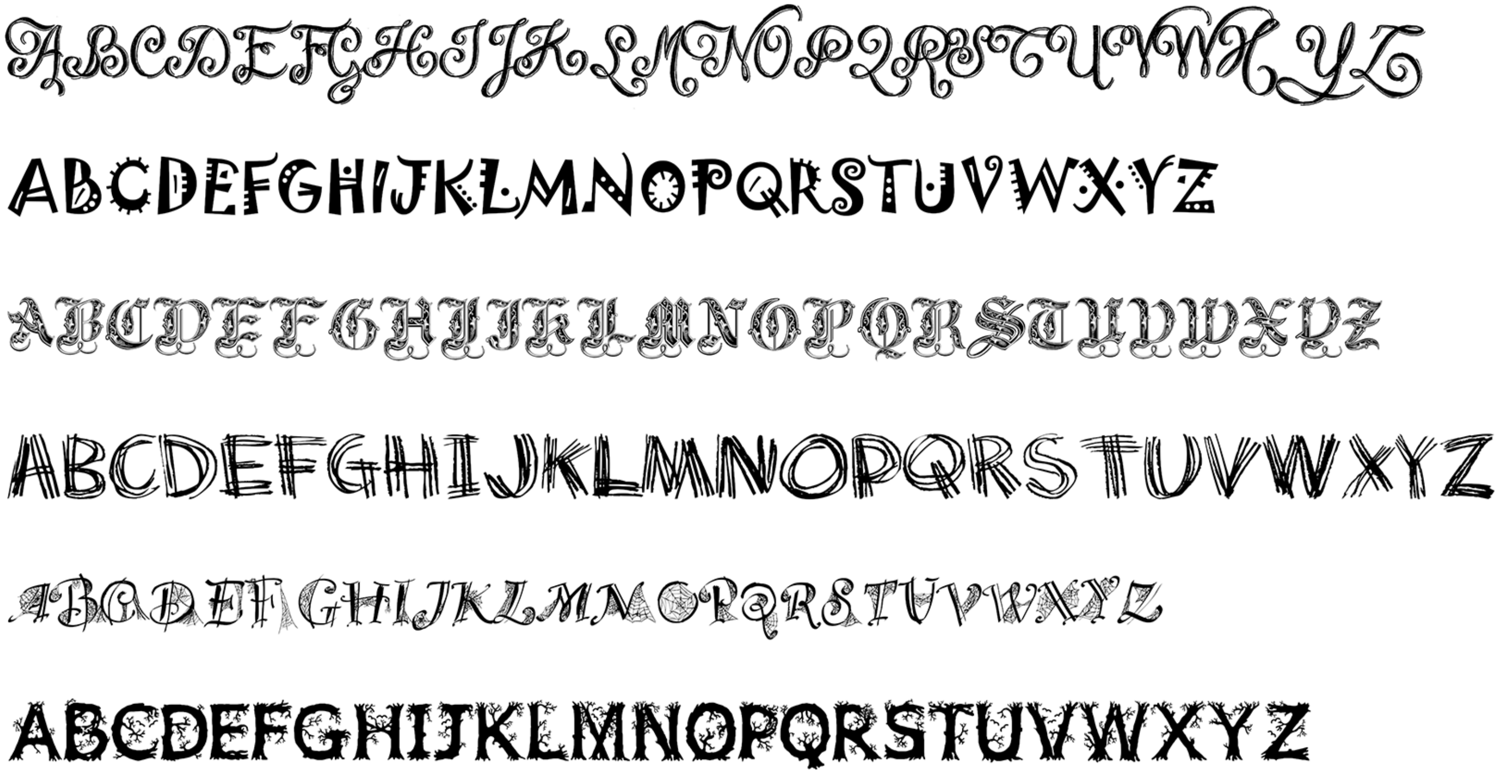


Results

- 4K display filled with text, timed on NV GeForce 1060

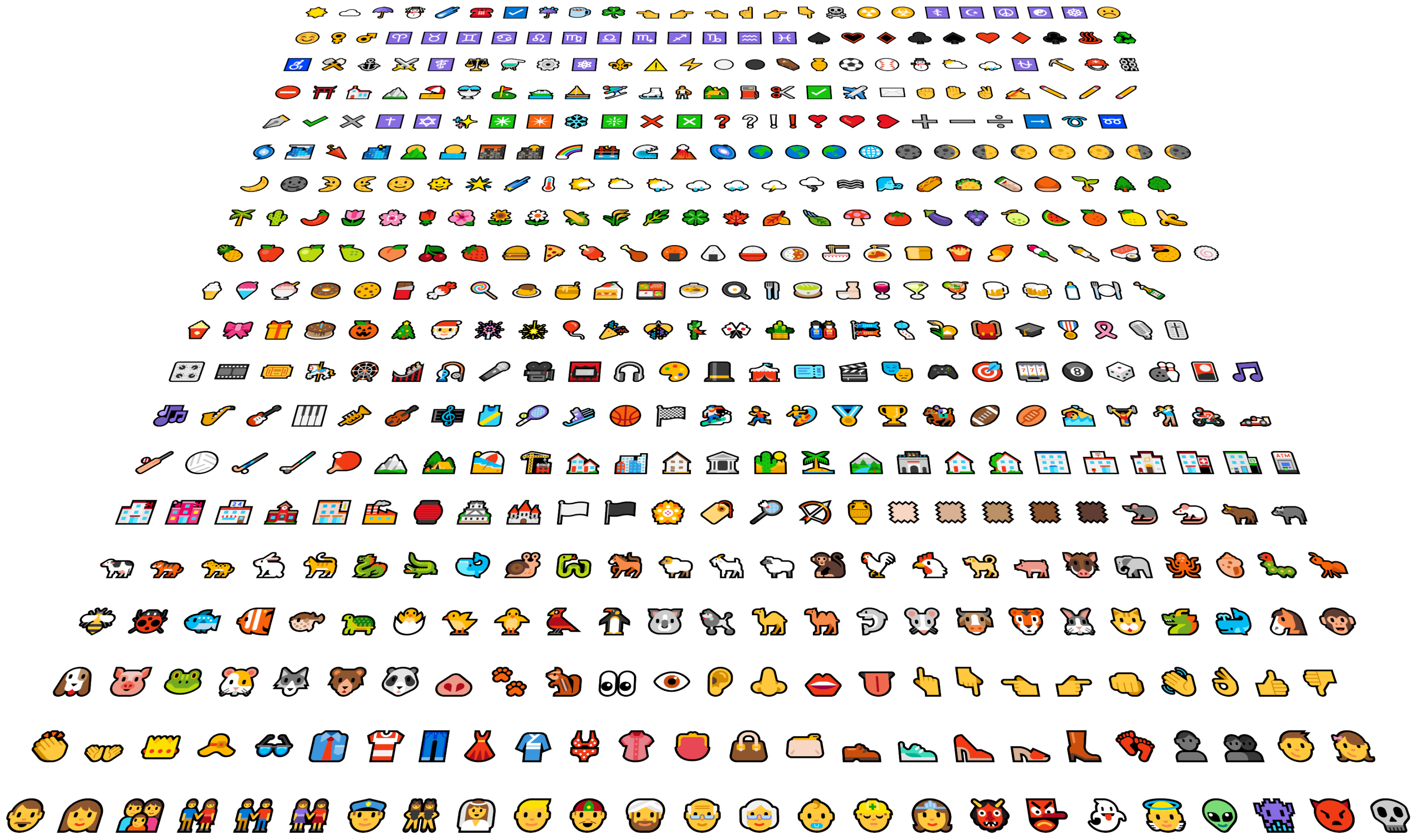
Font	Sample	Complexity	Time (ms)
Arial	ABCDEFG	28	0.70
Minion	ABCDEFG	35	0.71
Times	ABCDEFG	35	0.73
Jokerman	ABCDEFG	60	1.1
Spider	ABCDEFG	500	2.8

Results

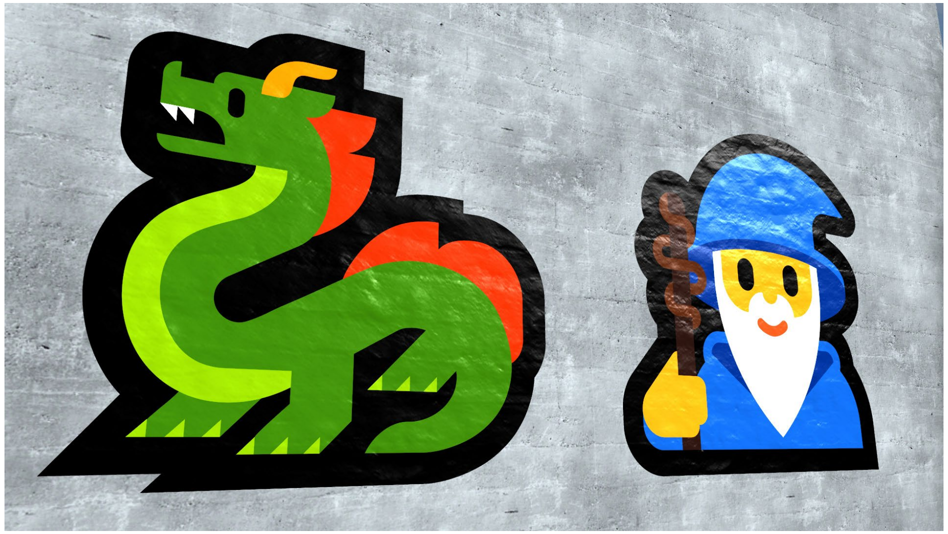

ABCDEFGHIJKLMNOPQRSTUVWXYZ
ABCDEFGHIJKLMNOPQRSTUVWXYZ
ABCDEFGHIJKLMNOPQRSTUVWXYZ
ABCDEFGHIJKLMNOPQRSTUVWXYZ
ABCDEFGHIJKLMNOPQRSTUVWXYZ
ABCDEFGHIJKLMNOPQRSTUVWXYZ

GPU-Centered Font Rendering Directly from Glyph Outlines

Results



Results



Questions?

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